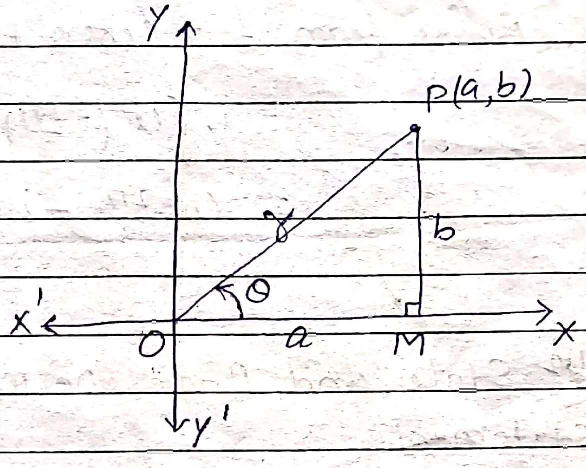
#1.1 Polar representation of Complex number :-

Let $z = a+ib$ be a complex number.

z is represented by $P(a,b)$ in the argand plane.



$\therefore OP = \sqrt{a^2+b^2} = |z| = r$

$\angle POM = \arg z = \theta$

In rt $\triangle OPM$,

$\cos\theta = \dfrac{a}{OP} = \dfrac{a}{|z|}$

$\therefore a = |z|\cos\theta$

And, $\sin\theta = \dfrac{b}{OP} = \dfrac{b}{|z|}$

$\therefore b = |z|\sin\theta$

Now, $z = a+ib$

$= |z|\cos\theta + i\,|z|\sin\theta$

$= |z|(\cos\theta + i\sin\theta)$

$\boxed{z = a+ib = r(\cos\theta + i\sin\theta)}$ where $r = |z|$ and $\theta = \arg(z)$

Also, In Euler form,

$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$

(we should take principal value of θ).

Note (i) $e^{i\theta} = \cos\theta + i\sin\theta$

(ii) $e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta$

(iii) $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

(iv) To remember :-

(a) $1 = \cos 0 + i\sin 0$ (b) $-1 = \cos\pi + i\sin\pi$

(c) $i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (d) $-i = \cos\pi - i\sin\pi$

Example : ① Write the polar form of $\frac{-1}{2} - \frac{i\sqrt{3}}{2}$.

Solution: Here, $z = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$, $a = -\frac{1}{2}$, $b = -\frac{\sqrt{3}}{2}$

$r = |z| = \sqrt{(\frac{-1}{2})^2 + (-\frac{\sqrt{3}}{2})^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$

$\therefore \tan\theta = \frac{b}{a} = \left|\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right| = \sqrt{3} = \tan\frac{\pi}{3}$

$\theta = \pi/3$

z lies in 3rd quadrant, $\arg(z) = -\pi + \pi/3.$

$= \frac{-3\pi+\pi}{3} = -\frac{2\pi}{3}$

Polar form $= r(\cos\theta + i\sin\theta)$

$= 1.\left\{\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right\}$

$= \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$

1.2 State and Prove De' moivre's Theorem. (Abraham De Moivres 1667–1754) French Mathematician

Statement :-

"If n be an integer, positive or negative, Then the value of $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$. and, If n be a fraction, positive or negative, then one of the values of $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$."

Proof:- Case I when n is a positive integer.

By Actual multiplication, we have

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$= \cos\theta_1 \cdot \cos\theta_2 + i\cos\theta_1 \cdot \sin\theta_2 + i\sin\theta_1 \cdot \cos\theta_2 + i^2 \sin\theta_1 \cdot \sin\theta_2$

$= (\cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2) + i(\cos\theta_1 \cdot \sin\theta_2 + \sin\theta_1 \cdot \cos\theta_2)$

$= \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$

Similarly,

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3)$

$= \cos(\theta_1 + \theta_2 + \theta_3) + i\sin(\theta_1 + \theta_2 + \theta_3)$

Proceeding in this way, the product of n factors,

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) \cdots (\cos\theta_n + i\sin\theta_n)$

$= \cos(\theta_1 + \theta_2 + \cdots + \theta_n) + i\sin(\theta_1 + \theta_2 + \cdots + \theta_n)$

Putting $\theta_1 = \theta_2 = \cdots = \theta_n = \theta$, we get

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Case II when n is a negative integer.

Let $n = -m$ so that m is a positive integer.

Then, $(\cos\theta + i\sin\theta)^n = (\cos\theta + i\sin\theta)^{-m}$

$$= \frac{1}{(\cos\theta + i\sin\theta)^m}$$

$$= \frac{1}{\cos m\theta + i\sin m\theta} \quad \{\text{By Demoivre's theorem Case I}\}$$

$$= \frac{1}{\cos m\theta + i\sin m\theta} \times \frac{\cos m\theta - i\sin m\theta}{\cos m\theta - i\sin m\theta}$$

$$= \frac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta + \sin^2 m\theta} \quad \{i^2 = -1\}$$

$$= \cos m\theta - i\sin m\theta$$

$$= \cos(-m)\theta + i\sin(-m)\theta \quad \{\cos(-\theta) = \cos\theta, \ \sin(-\theta) = -\sin\theta\}$$

$$= \cos n\theta + i\sin n\theta$$

Case III when n is a fraction, positive or negative.

Let $n = \frac{p}{q}$, where p is an integer either positive or negative and q is a positive integer.

Then,

$$\left(\cos p.\frac{\theta}{q} + i\sin p.\frac{\theta}{q}\right)^q = \cos\left(\frac{p\theta}{\cancel{q}}.\cancel{q}\right) + i\sin\left(\frac{p\theta}{\cancel{q}}.\cancel{q}\right)$$

$$= \cos p\theta + i\sin p\theta$$

$\therefore$ Taking the q^{th} root of both side, we get $\cos\frac{p\theta}{q} + i\sin\frac{p\theta}{q}$ is one of the q^{th} roots of $\cos p\theta + i\sin p\theta$

But $(\cos p\theta + i\sin p\theta) = (\cos\theta + i\sin\theta)^p$

$\therefore \cos\frac{p\theta}{q} + i\sin\frac{p\theta}{q}$ is one of the q^{th} root of $(\cos\theta + i\sin\theta)^p$

$\Rightarrow \cos\frac{p\theta}{q} + i\sin\frac{p\theta}{q}$ is one of the values of $(\cos\theta + i\sin\theta)^{p/q}$.

Therefore, One of the values of $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$

Example

① Simplify $(\sin\theta + i\cos\theta)^{2n}$.

Ans:- Now, $\sin\theta + i\cos\theta = i\left(\cos\theta + \frac{1}{i}\sin\theta\right)$

$= i\left(\cos\theta + \frac{i}{i} \times \frac{1}{i}\sin\theta\right)$

$= i(\cos\theta - i\sin\theta)$

$\therefore (\sin\theta + i\cos\theta)^{2n} = i^{2n} \cdot (\cos\theta - i\sin\theta)^{2n}$

$= (i^2)^n \cdot (\cos 2n\theta - i\sin 2n\theta)$

$= (-1)^n \cdot (\cos 2n\theta - i\sin 2n\theta)$

② Prove that $(1+i\sqrt{3})^8 + (1-i\sqrt{3})^8 = -256$

Ans:- Put $1 = r\cos\theta$ and $\sqrt{3} = r\sin\theta$

$\therefore r^2 = 1^2 + (\sqrt{3})^2 = 1+3 = 4 \Rightarrow r = 2$

And, $\tan\theta = \frac{\sqrt{3}}{1} = \tan\frac{\pi}{3} \Rightarrow \theta = \pi/3$

Now, $1+i\sqrt{3} = r(\cos\theta + i\sin\theta) = 2\cdot(\cos\pi/3 + i\sin\pi/3)$

$1-i\sqrt{3} = r(\cos\theta - i\sin\theta) = 2(\cos\pi/3 - i\sin\frac{\pi}{3})$

$LHS = (1+i\sqrt{3})^8 + (1-i\sqrt{3})^8$

$= \{r(\cos\theta + i\sin\theta)\}^8 + \{r(\cos\theta - i\sin\theta)\}^8$

$= r^8 \cdot (\cos 8\theta + i\sin 8\theta) + r^8 \cdot (\cos 8\theta - i\sin 8\theta)$

$= r^8 \cdot [\cos 8\theta + \cancel{i\sin 8\theta} + \cos 8\theta - \cancel{i\sin 8\theta}]$

$= 2^8 \times 2\cos(8 \times \pi/3)$

$= 2^9 \times \cos\left(2\pi + \frac{2\pi}{3}\right) = 512 \times \cos\frac{2\pi}{3}$

$= 512 \times \cos(\pi - \pi/3)$

$= 512 \times \left(-\cos\frac{\pi}{3}\right)$

$= \overset{256}{\cancel{512}} \times \left(-\frac{1}{\cancel{2}}\right) = -256 = RHS$

Real Analysis

① GRADUATION (4 Years Programme)

Choice-Based-Credit System (CBCS) (From 2023)

1st year		2nd year		3rd year		4th year	
SEM-I	SEM-II	SEM-3	SEM-4	SEM-5	SEM-6	SEM-7	SEM-8
MJC-01	02	03, 04	05, 06, 07	08, 09			

- Total Major Cource (MJC) :- 16
- Total Minor Cource (MIC) :- 10

~~Ability Enhancement Cource~~

- Total Multidisciplinary Cource (MDC) - 3
- Ability Enhancement Cource (AEC) - 2
- Skill Enhancement Cource - 3

~~Total~~

SEMESTER-1

S.N.	Name of cources	Types of Cource	L-T-P	Credit	Marks
1.	Major Cource - 1	MJC - 1	6-1-0	6	100
2.	Minor Cource - 1	MIC - 1	4-1-0	3	100
3.	Multi Disciplinary Cource-1	MDC-1	4-1-0	3	100
4.	MIL	AEC - 1	2-1-0	2	100
5.	Skill Enhancement Cource	SEC - 1	1-0-3	3	100
6.	Valued Added Cource	VAC-1	1-0-3	3	100

Total Credit = 20

Syllabus of BSc (Math) - Samester - 1

MJC - 1 (Major Cource)

② Semester-I MJC-01 : Algebra (6 credits) (Lecture: 60)

Unit-01 (Higher Trigonometry) - 10 lectures

1.1 # Polar representation of Complex numbers.
1.2 # De Moivres theorem and its application.
1.3 # Logarithms of complex quantities
1.4 # Hyperbolic functions
1.5 # Gregories series
1.6 # Summation of series
1.7 # Resolution in to factors.

Unit-02 (Set theory) - Lecture: 12

#2.1 Cartesian Product of sets
#2.2 Equivalence relation
#2.3 Partition
#2.4 Partial and total order relation functions
#2.5 Composition of functions
#2.6 Invertible function
2.7 Cardiality of a set.
2.8 Countable and Uncountable of a set.
2.9 Cantor's Theorem

Unit-03 (Number Theory)

3.1 well-ordering property of Integers.
3.2 Division Algorithm
3.3 Euclidean algorithm
3.4 Fundament theorem of Arithmetic
3.5 Modular arithmetic and basic properties of Congruences
3.6 Principle of mathematical Induction.

Unit-04 (Matrices) lecture : 12

#4.1 Matrices, operation on Matrices, Kinds of matrices

#4.2 Transpose, Symmetric & Skew Symmetric matrices

#4.3 Hermitian, skew Hermitian matrices

#4.4 Adjoint and Inverse of a matrix

#4.5 orthogonal matrices

#4.6 Solution of a System of linear equation by matrix method.

#4.7 Echelon forms

#4.8 Rank of a matrix

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Date

Unit-05 Theory of Equation

- Fundamental Theorem of Algebra
- Relation between roots and Coefficients of a Polynomial equation
- Symmetric function of roots
- Transformation of Equation
- Descarte's Rule of sign
- Solution of cubic equation (Cardon's Method)
- Biquadratic Equation (Euler's Method)